## I. *Inventio Curvæ quam Corpus descendens brevissimo tempore describeret; urgente Vi Centripetâ ad datum punctum tendente, quæ crescat vel decrescat juxta quamvis Potentiam distantiæ à Centro; dato nempe imo Curvæ puncto & altitudine in principio Casus. Per* Joh. Machin, Astron. Profess. Gresh. *&* Reg. Soc. Secret.

SIt centrum Virium C, (*Fig.* 1. 2), quo centro ad distantiam CB æqualem altitudini unde Corpus casurum est, describatur Circulus B E G, & fiat angulus B C G rectus. Ponatur A punctum Curvæ infimum, ubi axi C B occurrit ad datam distantiam C A. Oportet invenire punctum Q, ubi Curva celerrimi descensûs E Q A occurrit circulo Q F, ad datam aliam distantiam C F. Problema hoc duos habet Casus, quorum alter pendet ab Hyperbola & Circulo, alter ab Ellipsi & Circulo.

*Cas.* 1. Si fuerit Vis centripeta reciproce ut distantia à Centro. Sit K L M (*Fig.* 1.) Hyperbola quævis rectangula centro C & Asymptoto C B descripta, quæ occurrat normalibus B K, A M super ipsam B C ad puncta B, A erectis, in K & M; ordinatæ vero cuilibet intermediæ F L ad punctum F erectæ, in L. Fiat C D ad C G ut $\surd$ A F L M ad $\surd$ A B K M, & sit D H normalis super CG: dein capiatur Sector R C B ad Aream H D C B ut data Area Hyperbolica A B K M ad datum Rectangulum C A × A M. Tum recta R C occurret circulo F Q in puncto Q, quod quidem est ad Curvam celerrimi descensus E Q A.

Hab-

Habebitur autem punctum E, à quo inciperet Corporis casus, capiendo Sectorem B C E ad Aream Quadrantis B C G, in eadem ratione Areæ Hyperbolicæ A B K M ad rectangulum sub C A & A M contentum.

*Coroll.* Hinc si recta R C, circa centrum C revoluta, faciat Sectores R C B proportionales Areis H D C B, in quibus quadrata Basium C D sumuntur in progressione Arithmeticâ: tum rectæ C R intersecabunt Curvam E Q A ad distantias à centro C Q, quæ decrescant in progressione Geometricâ

*Cas.* 2. Si vero Vis centripeta fuerit reciproce ut alia quævis Potestas distantiæ à centro; sit $n + 1$ Index istius Potestatis (ubi $n$ potest esse Numerus quilibet integer vel fractus, affirmativus vel negativus) sitque $H = CB$ altitudo maxima Curvæ quæsitæ E Q A, $h = CA$ altitudo minima ejusdem, & $A = CF$ altitudo alia quævis intermedia. *Fig.* 2.

In recta C G capiatur C D ad C B ut $\sqrt{h^n}$ ad $\sqrt{H^n}$, atque etiam C H ad C D ut $\sqrt{A^n - h^n}$ ad $\sqrt{H^n - h^n}$. Centro C, semiaxibus C D, C B, describatur Ellipsis B L D, cui occurrat ordinatim applicata H L in puncto L; & ducatur recta L K, quæ Ellipsin tangat in L, & Axi minori C D producto conveniat in K: dein Tangenti K L parallela ducatur N M, circulum B E M G tangens in M & ipsi C D occurrens in N. Denique capiatur Sector R C B, qui sit ad Aream N M B L K N, inter Circulum & Ellipsin & utriusque Tangentes rectamque N K comprehensam, in ratione Numeri binarii ad Numerum $n$. Tum recta R C intersecabit Circulum F Q in puncto Q, quod erit ad Curvam celerrimi Descensus E Q A.

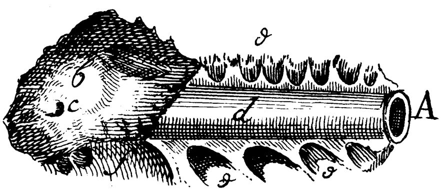
Quod si fiat Sector B C E ad aream B D G, inter Ellipseos & Circuli Quadrantes interceptam, in ratione dictâ Binarii ad Numerum $n$, coeuntibus scilicet punctis L, D & M, G; (ob $A^n = H^n$) erit punctum E unde inchoa-

choaretur Caſus Corporis breviſſimo tempore deſcendentis ad A, deſcenſuque ſuo Curvam E Q A deſcribentis, quam tangit recta C E in E, quamque ad angulos rectos ſecat C B in A.

Harum Conſtructionum Demonſtrationes è Celeberrimi D. *Newtoni Quadraturis*, ejuſdemque *Philoſ. Nat. Principiis* (Prop. XXXIX. & ſequentibus aliquibus) petitæ, aliâ datâ occaſione oſtendentur. Problema autem eſt alterius generis, Deſcribere Curvas per quas Corpora, de puncto ſummo E, ſeu principio caſus, demiſſa, celerrimo deſcenſu ad inferiora data puncta Q, urgente qualibet Vi centripeta, ferrentur; cujus quidem ſolutio in poteſtate eſt. In præſentia ſufficiat generalem hujuſmodi Curvarum tradidiſſe Ideam, earumque ad Circuli & Hyperbolæ Quadraturas relationes indicaſſe, abſque quibus eaſdem Geometrice conſtruere haud adeo proclive eſt.

II. *De*

Fig, 1,




Fig, 11.

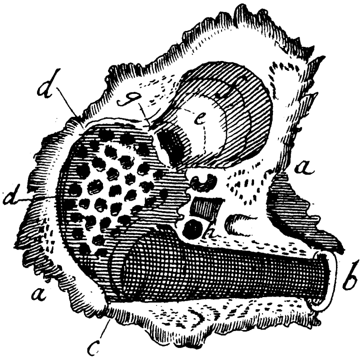



Fig. III

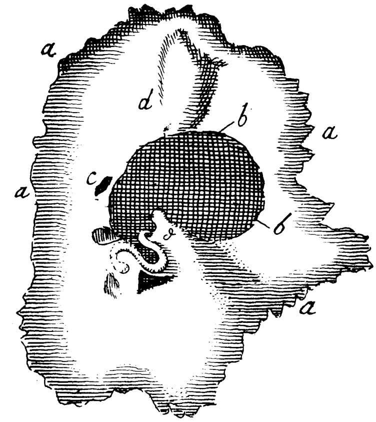
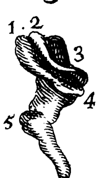


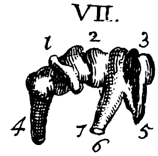

Fig. IIII



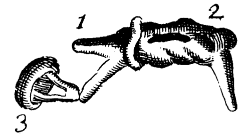

V


VI

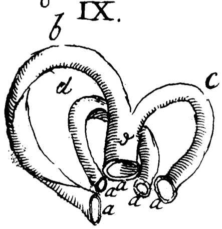

VII.




VIII.




IX.




X

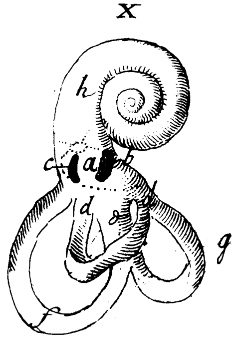
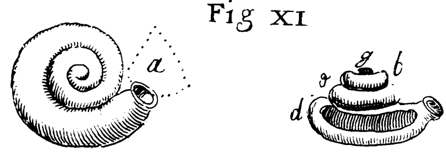



Fig XI



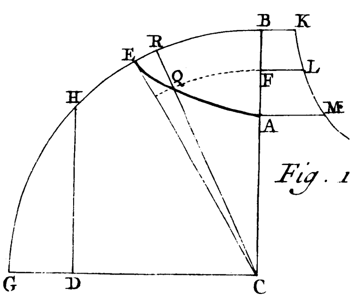




*Fig. 1.*

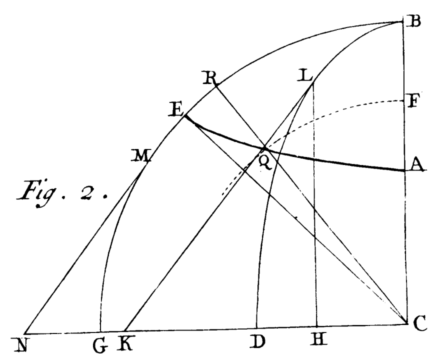



*Fig. 2.*